Dana Center **Mathematics** PATHWAYS

Creating a Scalable and Contextualized Mathematics Pathway to Prepare Science and Engineering Students for Success in Calculus

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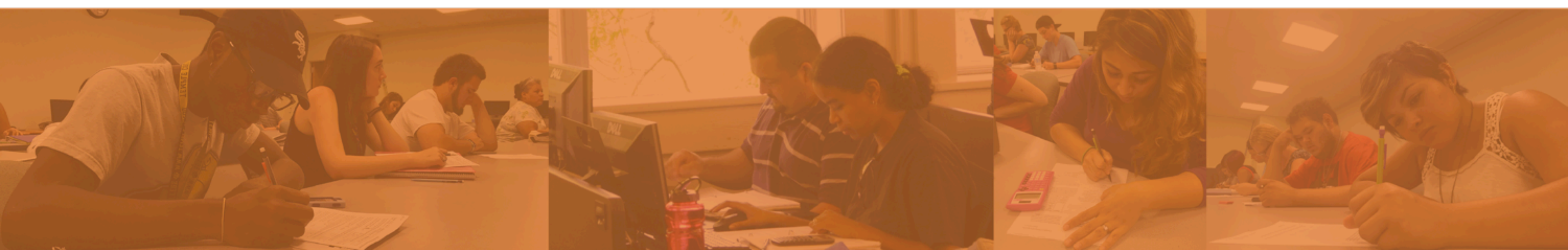

Outcomes

1) Gain an understanding of math pathways from a national perspective

2) Identify opportunities to leverage math pathways to broaden participation in STEM.

3) Understand the role partner disciplines play in scaling Math Pathways

4) Explore content and structure considerations for preparing students for calculus

DCMP Vision

Dana Center Mathematics PATHWAYS
dcmathpathways.org

All students have equitable access to and the opportunity for success in rigorous mathematics pathways that are aligned and relevant to their future aspirations, propelling them to upward economic and social mobility.

The DCMP seeks to ensure that **ALL** students in higher education will be:

- **Prepared** to use mathematical and quantitative reasoning skills in their careers and personal lives,
- **Enabled** to make timely progress towards completion of a certificate or degree, and
- **Supported** and **Empowered** as mathematical learners.

Definition of *math pathway*

. . . a mathematics course or sequence of courses that students take to meet the requirements of their program of study.

The concept of math pathways applies to *all* students.

Dana Center Principles for Pathways

Mathematics pathways are structured so that:

1) All students, regardless of college readiness, enter directly into mathematics pathways **aligned** to their programs of study.

2) Students complete their first college-level math requirement in their first year of college.

Dana Center Principles for Pathways

Students engage in a high-quality learning experience in math pathways designed so that:

3) Strategies to support students as learners are integrated into courses and are aligned across the institution.

4) Instruction incorporates evidence-based curriculum and pedagogy.

Dana Center Principles for Pathways

Quick structural change

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Continuous improvement

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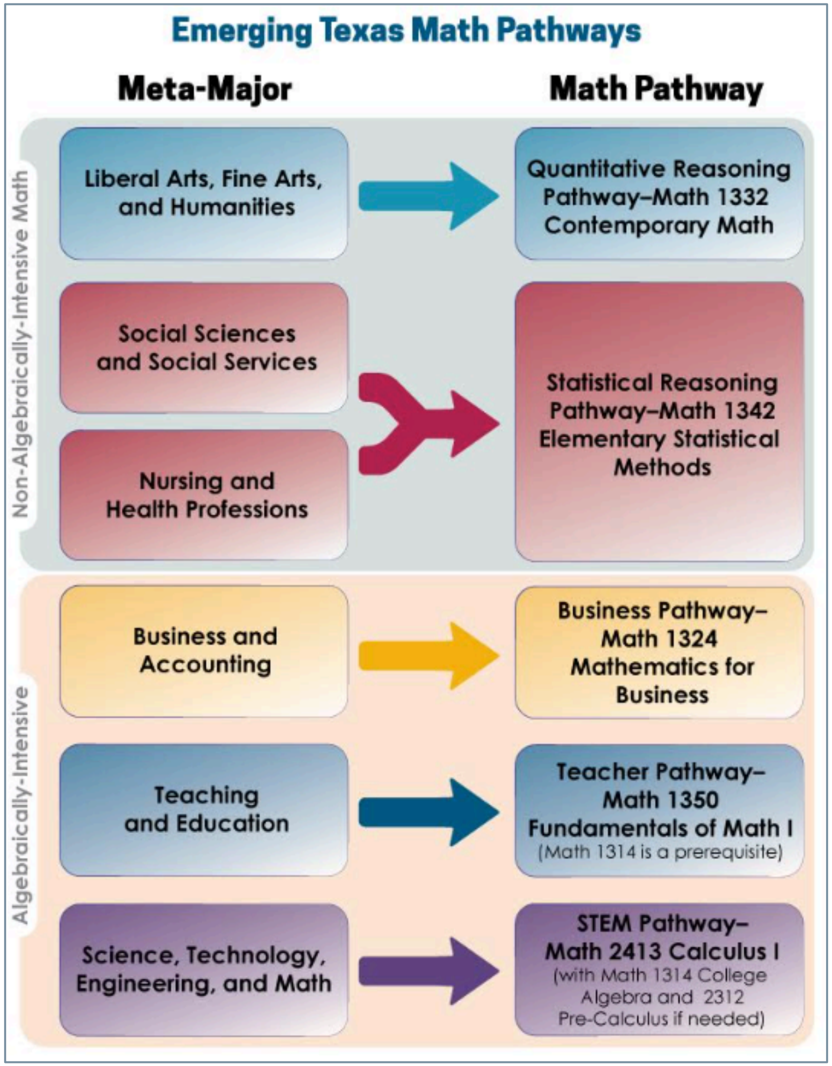
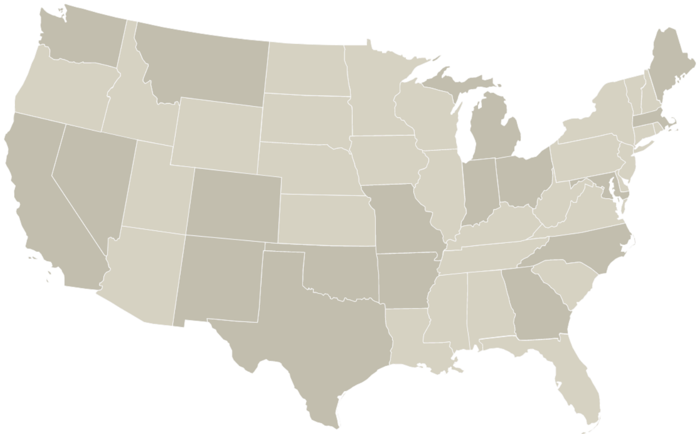

Emerging Texas Math Pathways
Meta-Major
Math Pathway
Non-Algebraically-Intensive Math
Liberal Arts, Fine Arts, and Humanities
Quantitative Reasoning Pathway–Math 1332 Contemporary Math
Social Sciences and Social Services
Nursing and Health Professions
Statistical Reasoning Pathway–Math 1342 Elementary Statistical Methods
Algebraically-Intensive
Business and Accounting
Business Pathway–Math 1324 Mathematics for Business
Teaching and Education
Teacher Pathway–Math 1350 Fundamentals of Math I
(Math 1314 is a prerequisite)
Science, Technology, Engineering, and Math
STEM Pathway–Math 2413 Calculus I
(with Math 1314 College Algebra and 2312 Pre-Calculus if needed)

National Work

Working in states and regions across the country…

Arkansas
California State System
Central Valley, California
Colorado
Georgia
Indiana
Maine
Massachusetts
Michigan
Missouri
Montana
Nevada
North Carolina System
Ohio
Oklahoma
Texas
Washington



Broad-based support

Growing momentum to modernize entry-level mathematics programs through pathways:

- Endorsed by major mathematics professional associations.
- Endorsed by major higher education policy and advocacy organizations.
- Increasingly implemented and supported in policy.

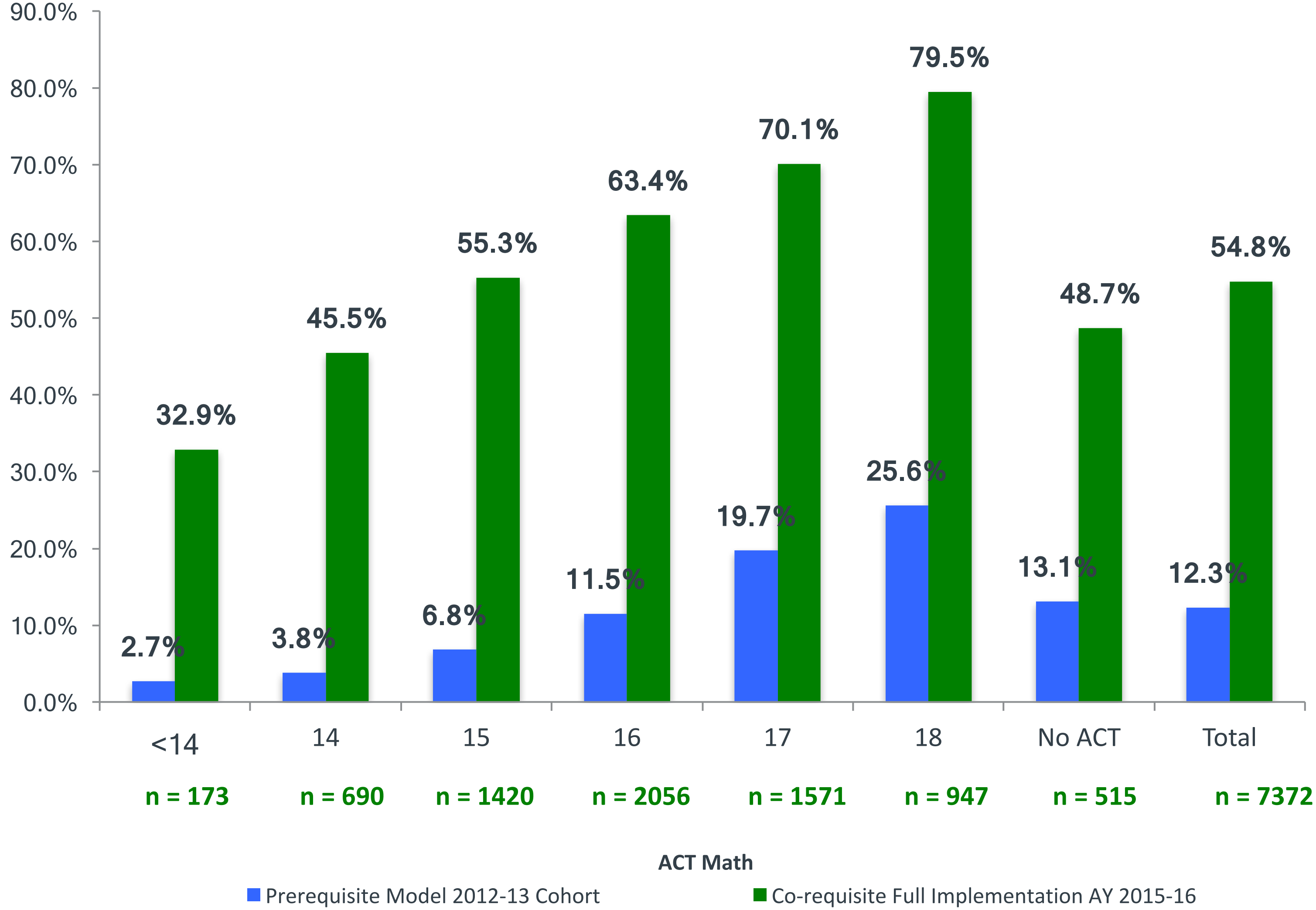
How are students doing?

Pathways data

When compared to traditional community college success rates for students placed two levels down, math pathway students experienced triple the success rates in half the time.

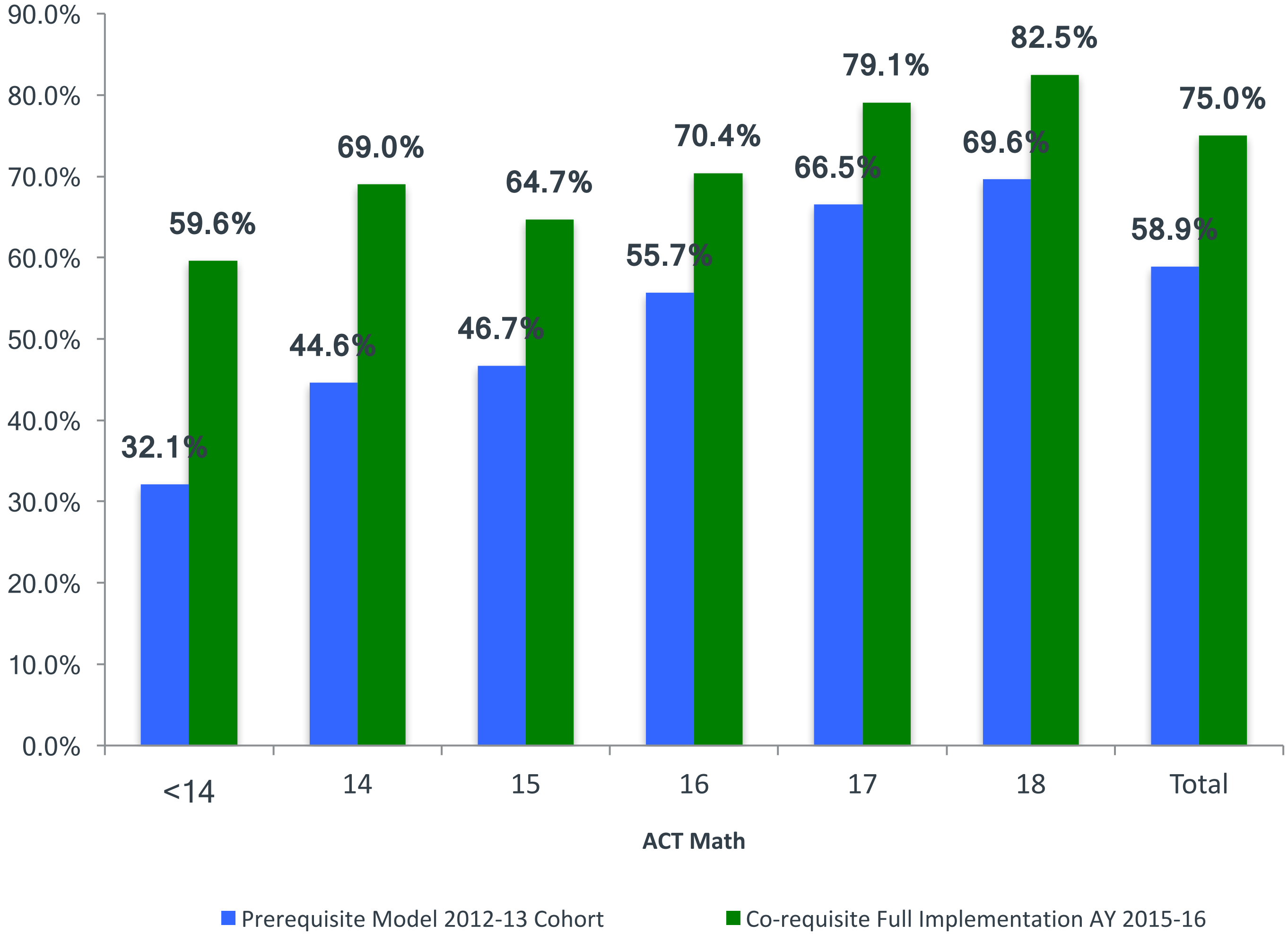
- Dana Center Math Pathways curriculum
- Carnegie Math Pathways curriculum

Tennessee Community Colleges Gateway Math Success in One Year



Tennessee Board of Regents Brief #3: *Co-Requisite Remediation Full Implementation 2015-16*

Tennessee Universities
Gateway Math Success in One Year



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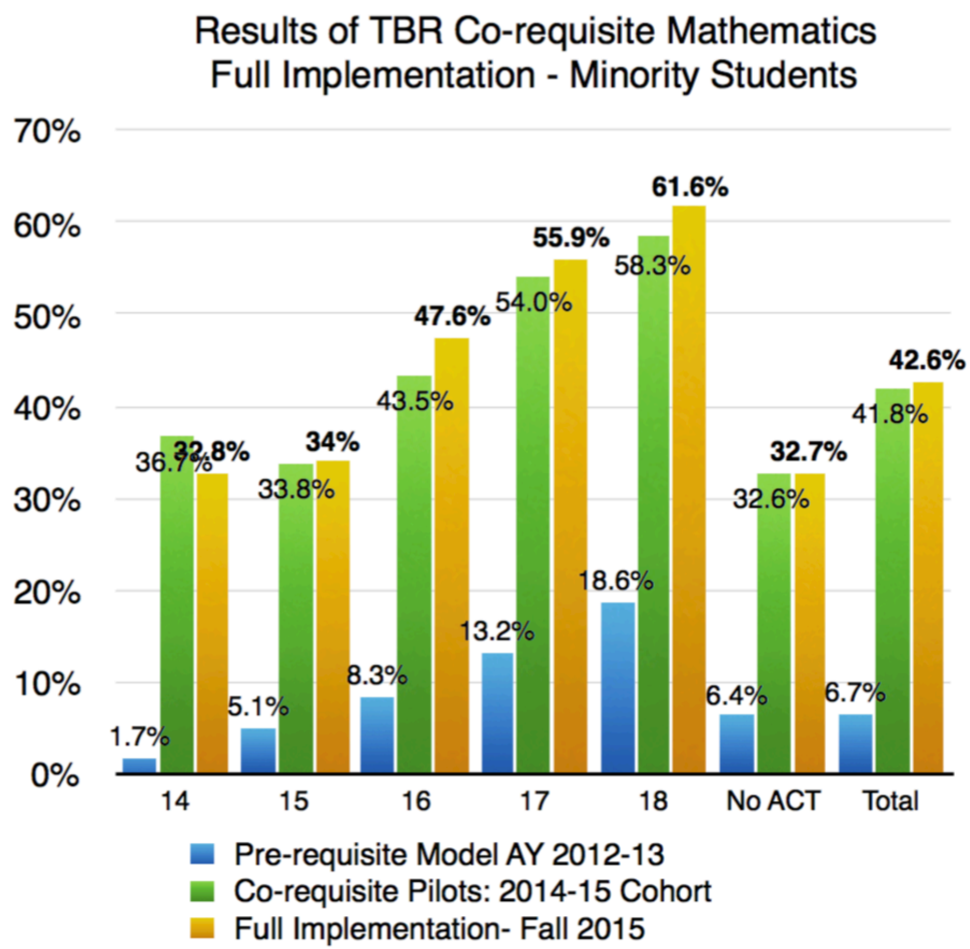
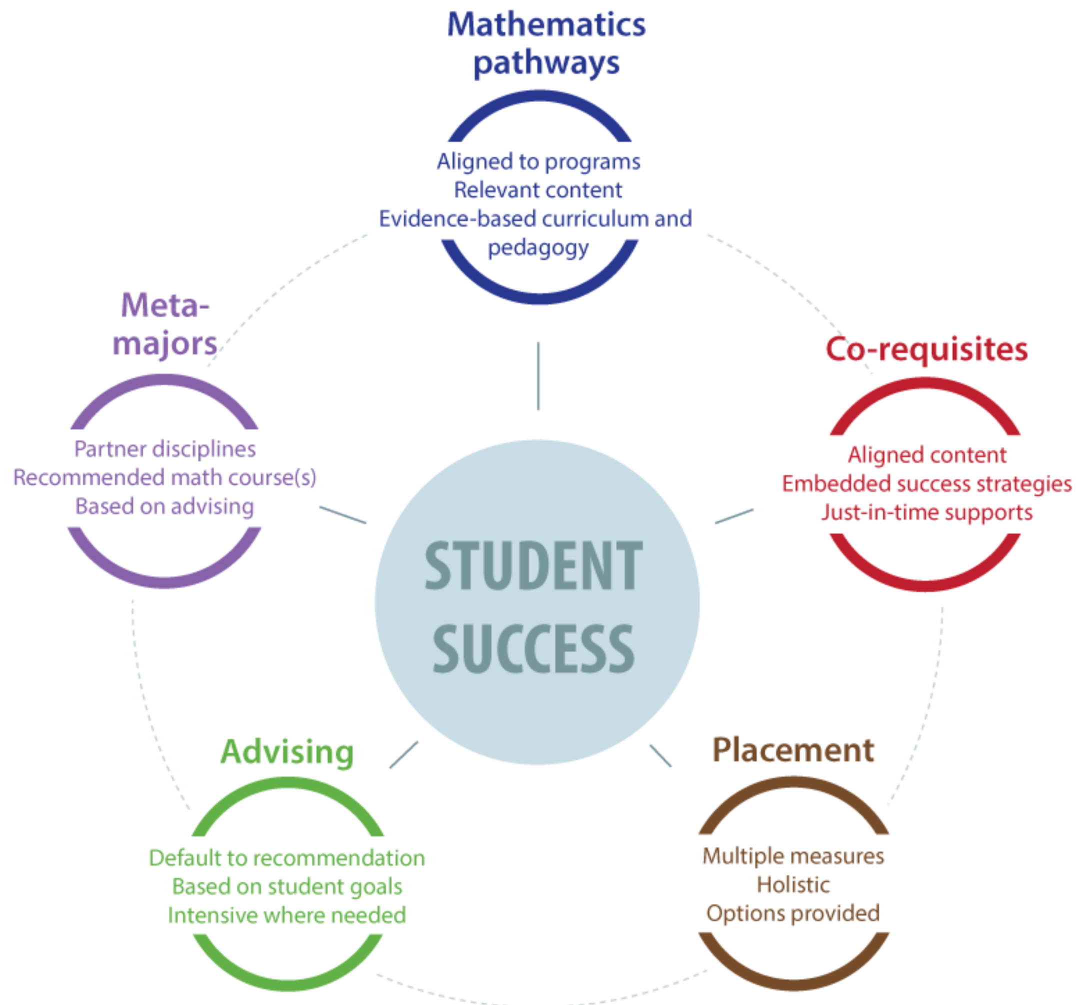
Equity


Results of TBR Co-requisite Mathematics
Full Implementation - Minority Students
70%
60%
50%
40%
30%
20%
10%
0%
1.7%
36.7%
32.8%
5.1%
33.8%
34%
8.3%
43.5%
47.6%
13.2%
54.0%
55.9%
18.6%
58.3%
61.6%
6.4%
32.6%
32.7%
6.7%
41.8%
42.6%
14
15
16
17
18
No ACT
Total
Pre-requisite Model AY 2012-13
Co-requisite Pilots: 2014-15 Cohort
Full Implementation- Fall 2015

Comprehensive Redesign



What about STEM?

Implementation of multiple math pathways offers an opportunity to broaden participation in STEM

HOW?

Discussion

Consider your institution for a moment:

Are you implementing math pathways?

If so, how is it going?

5
min

Discussion

Consider your institution for a moment:

Are math pathways aligned at scale?

How do you know?

5 min

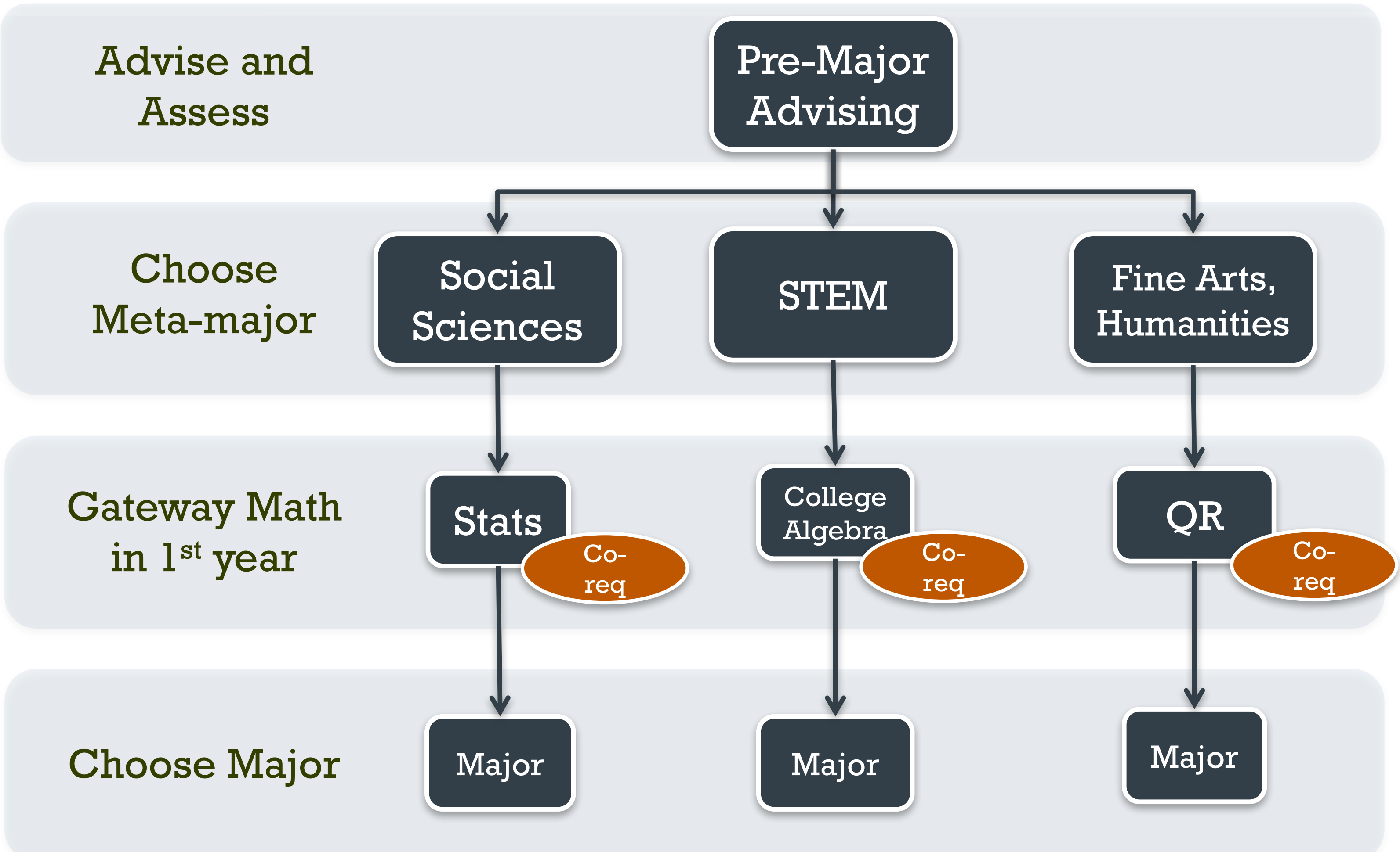
Discussion

Consider your institution for a moment:

What role do other disciplines play in aligning math pathways to programs of study?

5 min

Mathematics Pathways with Co-Requisites



Adapted from Complete College America 2016

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3) Strategies to support students as learners are integrated into courses and are aligned across the institution.
4) Instruction incorporates evidence-based curriculum and pedagogy.

A Challenge............

- STEM occupations are expected to grow at a rate 1.4 times faster than non-STEM occupations (Noonan, 2017)
- The U.S. will need approximately one million more STEM professionals between 2014 and 2024 (Noonan, 2017)

But…

- Fewer college students are truly ready for college-level mathematics.
- Poor success rates in developmental mathematics courses are negatively impacting the number of STEM degrees awarded (Kreysa, 2006)

An Opportunity…………

Implementation of multiple math pathways presents an opportunity to reenvision the pathway to Calculus.

What role could STEM disciplines and STEM professionals have in this process?

Process for reenvisioning the pathway to Calculus

Content

- Utilize a back-mapping process: What are the difficult concepts/ideas students encounter in Calculus?
- Create true prerequisite courses that prepare students for Calculus instead of developing a list of favorite topics or settle for semester after semester of developmental level algebra courses.

Architecture

- Consider the elements of instruction we know help students. For example:
  - Active and collaborative learning.
  - Meaningful experiences through contextualized mathematics.
  - Opportunities for struggle and perseverance.

Student struggle in Calculus

What are the difficult ideas/concepts students encounter in Calculus?

- Functions and function notation
- Algebra
- Concepts of inverse functions and function composition
- Communicating about change and rates of change
- Limits and approximations
- The integral as an accumulator
- Dynamic geometric reasoning
- Problem solving skills: perseverance, drawing pictures, creating equations
- Working with open-ended problem structures

Four overarching principles (from DCMP work)

1. Deep understanding of the function process (contrasted with an action view of a function)

2. Ability to apply *covariational reasoning*

3. Ability to communicate with functions and use function notation

4. Meaningful approaches to the development of algebraic reasoning

Student views of functions

Think about a precalculus course you or a colleague taught recently.

Think about how a reasonably good student might describe "functions" after leaving the course. How would they answer these two questions:

What are functions?

What are they used for?

Student views of functions

Action view of a function

- Students see functions as static.
- Computations involve evaluating a function at a single point.

Process view of a function

- Functions are processes that can be composed and inverted.
- Functions are processes that take a continuum of input values and produces a continuum of output values. i.e. Functions can be studied by examining their behavior on intervals of input values.
- Functions are used to model dynamic situations.

Reference: **Marilyn Carlson, Michael Oehrtman, and Nicole Engelke** (2010), "The precalculus concept assessment: a tool or assessing students' reasoning abilities and understandings", Cognition and Instruction, 28:2, 113-145.

Lack of developing a process view results in:

- Difficulty composing and inverting functions
- Inability to use functions effectively in word problems
- Graphs of functions are viewed as fixed curves [not as a representation of a mapping of input/output values].
- Points on graphs and slopes are viewed as [fixed] geometric properties of graphs not as properties of the underlying functions.
- Conflating the shape of a graph with the physical situation being modeled.
- Inability to interpret or express contextual relationships
- Inability to use symbols meaningfully [e.g. constructing algebraic formulas to represent relationships]

Change is central to Calculus

Think about a precalculus course you've recently taught…

- Do you think students left feeling that the idea of measuring and describing change was central to the course? Explain.

Covariational Reasoning

Considering how one variable changes while imagining changes in the other variable.

Five Levels of Covariational Reasoning

- Recognizing that one variable depends on another
- Identifying the direction of change of one variable as the second variable changes by a given amount
- Identifying the amount of change of one variable
- Identifying an average rate of change
- Attending to the change in the rate of change – instantaneous rate of change

These "levels" are a great help for designing content!

Reference: **Marilyn Carlson, Michael Oehrtman, and Nicole Engelke** (2010), "The precalculus concept assessment: a tool or assessing students' reasoning abilities and understandings", Cognition and Instruction, 28:2, 113-145.

In terms of content this means:

Explore concepts with multiple representations.

Functions are processes not algebraic formulas.

Describe behavior of functions on entire intervals.

Develop the language and inclination to describe how one quantity changes with respect to another: increasing/decreasing, rates of change, average rate of change, move towards instantaneous rate of change.

In terms of content this means:

Practice dynamical reasoning (imagine running through many input/output combinations without actually performing all of them).

Create an engaging curriculum set within authentic STEM contexts and models (exploring, creating, and interpreting mathematical models will drive the algebraic development).

Build in frequent opportunities for students to practice communicating both orally and in writing.

Make the algebra Meaningful! Give the procedures meaning!

What could this look like?

- Sample activities encompass a 25 minute learning episode
- Each activity is completed in small groups with instructor facilitation

Biological Populations

- Early in a course
- Exploring different function types, mastering use of function notation and how to communicate with functions

Note: Use of multiple representations, communication elements, average rate of change, describing change over intervals, working with covarying quantities.

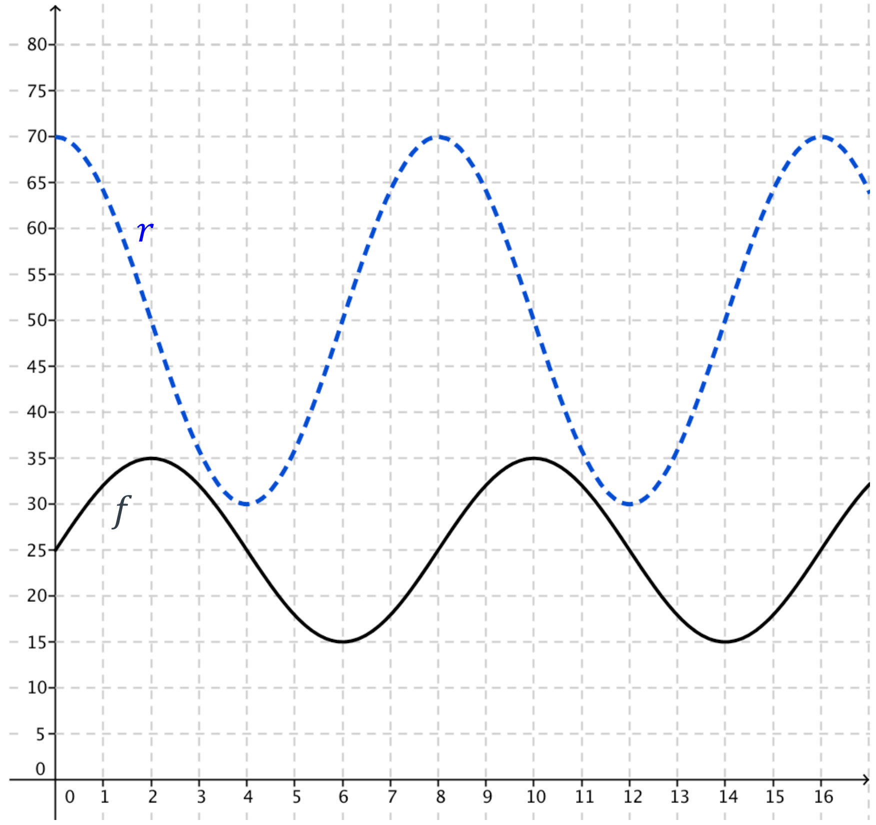
Let $r(t)$ represent the number of rabbits and $f(t)$ represent the number of foxes in a forest, where t is the number of years after 1990. The graphs of r and f appear below. Use these graphs to answer the following questions.

2) Use the graph of r to answer these questions:

Part A: Why does the function r appear to be periodic? State its period, using correct units.

Part B: Assuming the same pattern continues, what is the rabbit population in 2010?

Part C: The rabbit population is increasing between 1994 and 1998. List the next three time intervals over which the rabbit population will be increasing.



4) How are the fox and rabbit populations changing between 1990 and 1992? Why might this be the case?

5) Between 1996 and 1998, both the rabbit and fox populations are increasing.

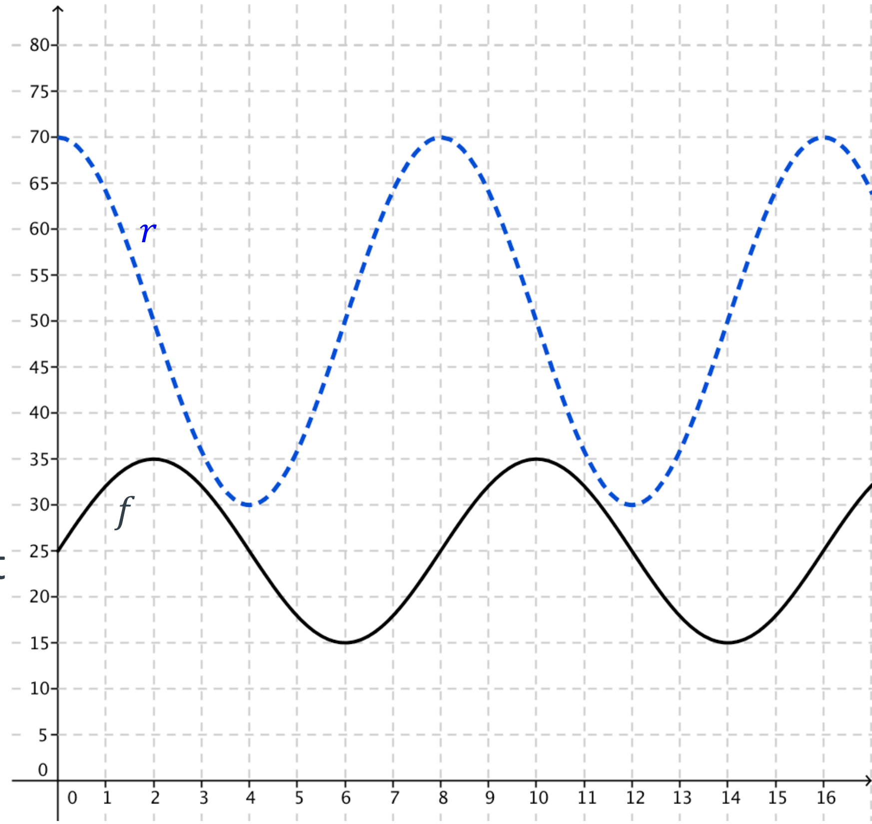
Part A: Find the average rate of change of the rabbit population. Be sure to use correct units.

Part B: What is the average rate of change of the fox population?

6) Use the graphs to calculate the following quantities, and write sentences interpreting what each quantity represents. Use appropriate units.

Part A: $r(10) - r(7)$

Part B: $\dfrac{f(6)-f(2)}{4yr}$

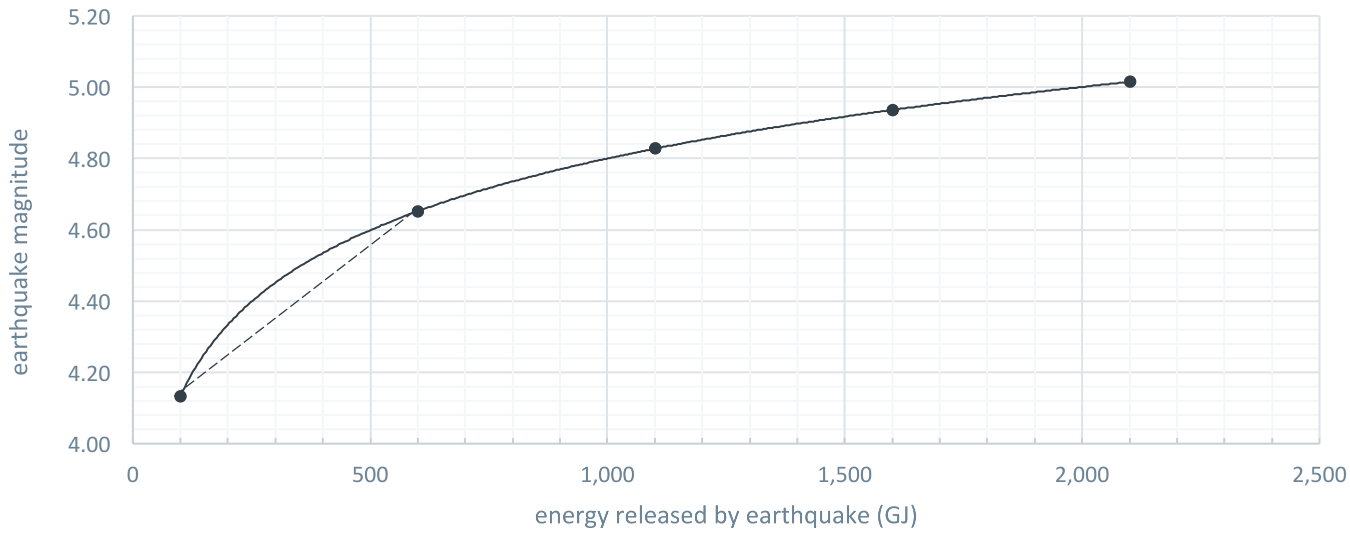


Geology

- Early in a course
- Exploring different function types

Note: Use of multiple representations, average rate of change connected to slope of secant line, verbal explanations of change, examining change over intervals, experiencing the rate of change of a function as a function itself.

Let’s continue investigating the earthquake magnitude scale that we started in the preview assignment. Here is the data table that we were working with, along with a plot of the magnitude vs. energy released.



energy released - in gigajoules (GJ)	earthquake magnitude	change in magnitude	average rate of change of magnitude
100	4.13	—	—
600	4.65	4.65 - 4.13 = 0.52	$= 0.52/500 = 1.04 \times 10^{-3}$
1100	4.83	0.18	
1600	4.94	0.11	
2100	5.01	0.08	

2) In the table above, calculate the average rate of change of the earthquake magnitude for each interval. Note that the first value has already been calculated for you: 1.04×10^{-3} is the average rate of change of earthquake magnitude as the energy released changes from 100 GJ to 600 GJ. Write your answers in scientific notation.

3) The graph above shows a (dashed) line segment that connects the first two points on the graph. The first point corresponds to an earthquake which releases 100 GJ of energy and the second point corresponds to an earthquake which releases 600 GJ of energy.

Part A: Calculate the slope of this line segment.

Part B: Draw another line segment connecting the second and third point on the graph and calculated its slope.

Part C: How do the slopes of these two lines compare with the average rate of change of earthquake magnitude?

Part D: If you were to draw a third line segment connecting the third and fourth point on the graph what would its slope be?

4) Write a sentence that explains how the average rate of change of the magnitude of an earthquake changes as the energy increases in equal amounts.

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Discussion

How do these ideas impact your current work?

5
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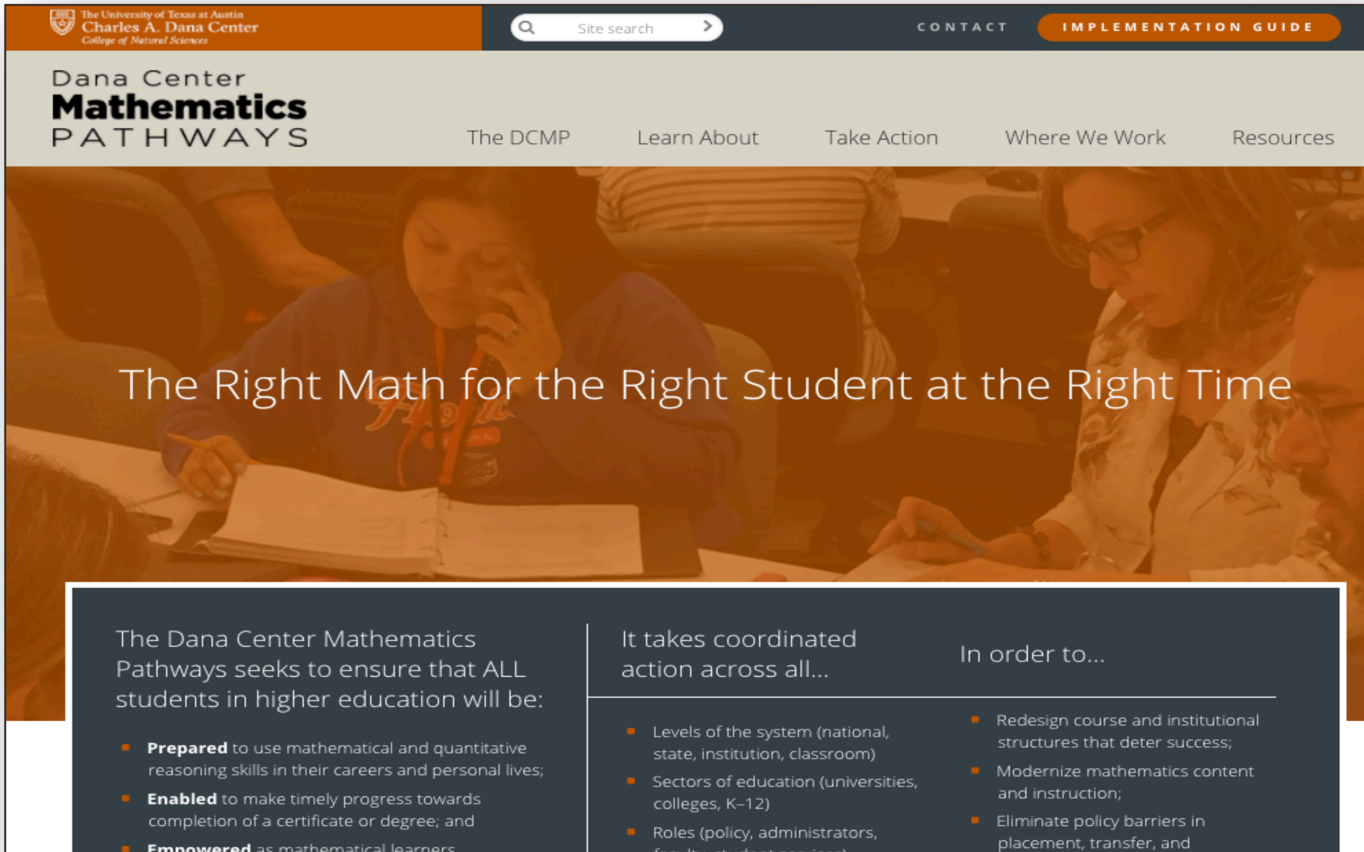
DCMP Resource Site

http://www.dcmathpathways.org/



Contact Information

- Frank Savina: fsavina@austin.utexas.edu

- General information about the Dana Center
  www.utdanacenter.org

- The DCMP Resource Site
  www.dcmathpathways.org

- To receive monthly updates about the DCMP, contact us at
  http://tinyurl.com/DanaCenterInBrief

About the Dana Center

The **Charles A. Dana Center** at The University of Texas at Austin works with our nation's education systems to ensure that every student leaves school prepared for success in postsecondary education and the contemporary workplace.

Our work, based on research and two decades of experience, focuses on K–16 mathematics and science education with an emphasis on strategies for improving student engagement, motivation, persistence, and achievement.

We develop innovative curricula, tools, protocols, and instructional supports and deliver powerful instructional and leadership development.

2016

The University of Texas at Austin
Charles A. Dana Center